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Kinematics 运动学

Displacement and Velocity 位移和速度：

Motion in 1 dimensions 一维运动	
初等数学	高等数学
Average velocity 平均速度： Position 位置： x Displacement 位移： $\Delta x = x_2 - x_1$ $v_{ave} = \bar{v} = \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{\Delta x}{\Delta t}$ **Instantaneous velocity 瞬时速度：** $v = \lim\limits_{\Delta t \to 0} \bar{v} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t}$	**Average velocity 平均速度：** 同初等数学 **Instantaneous velocity 瞬时速度：** $v = \lim\limits_{\Delta t \to 0} \bar{v} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t} = \dfrac{dx}{dt}$ $dx = vdt \quad or \quad \int_{x0}^{x} dx = \int_{0}^{t} v\,dt$
Motion in 2 dimensions 二维运动	
初等数学	高等数学
Position 位置： $\vec{r} = \vec{r}(t) = x(t)\vec{\imath} + y(t)\vec{\jmath} \quad \begin{cases} x = x(t) \\ y = y(t) \end{cases}$ **Displacement 位移：** $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$ $= (x_2 - x_1)\vec{\imath} + (y_2 - y_1)\vec{\jmath}$ **Average velocity 平均速度：** $\vec{v}_{ave} = \dfrac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \dfrac{\Delta\vec{r}}{\Delta t}$ **Instantaneous velocity 瞬时速度：** $\vec{v} = v_x\vec{\imath} + v_y\vec{\jmath} = \lim\limits_{\Delta t \to 0} \vec{v}_{ave} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta\vec{r}}{\Delta t}$	**Average velocity 平均速度：** 同初等数学 **Instantaneous velocity 瞬时速度：** $\vec{v} = v_x\vec{\imath} + v_y\vec{\jmath} = \lim\limits_{\Delta t \to 0} \vec{v}_{ave}$ $= \lim\limits_{\Delta t \to 0} \dfrac{\Delta\vec{r}}{\Delta t} = \dfrac{d\vec{r}}{dt}$ $\begin{cases} v_x = dx/dt \\ v_y = dy/dt \end{cases} \qquad \begin{cases} dx = v_x dt \\ dy = v_y dt \end{cases}$ $\begin{cases} \int_{x0}^{x} dx = \int_{0}^{t} v_x\,dt \\ \int_{y0}^{y} dy = \int_{0}^{t} v_y\,dt \end{cases}$

Example: An object moves in the xy-plane with a velocity given by

$$\vec{v}(t) = 3t^2\vec{\imath} + 4\sin 2t\,\vec{\jmath}$$

where x and y are in meters and v is in meters per second. What is its displacement between $\mathrm{t} = 0\mathrm{s}$ and $\mathrm{t} = 2\mathrm{s}$?

Solution let $\vec{r}$ be the displacement in meters:

$$\int_{r0}^{r} d\vec{r} = \int_0^2 \vec{v}\,dt = \int_0^2 (3t^2\vec{\imath} + 4\sin 2t\,\vec{\jmath})\,dt$$

$$\Rightarrow \vec{r} - \vec{r}_0 = \left(t^3\vec{\imath} - 2\cos 2t\,\vec{\jmath}\right)_0^2 = 8\vec{\imath} - (2\cos 4 - 2)\vec{\jmath} = \Delta\vec{r}$$

Example: A projectile is fired up an incline (incline angle ϕ) with an initial speed $\vec{v}$ at an angle θ with respect to the horizontal ($\theta > \phi$) as shown in the figure.

(a) Find the time t when the project hit the incline.

In the horizontal direction $v_x(t) = v\cos\theta + a_x t = v\cos\theta \quad (a_x = 0)$
In the vertical direction $v_y(t) = v\sin\theta + a_y t = v\sin\theta - gt \quad (a_y = -g)$
The displacement in the horizontal(vertical) direction are $x(y)$:

$$\int_0^x dx = \int_0^t v_x\,dt \Rightarrow x - 0 = \int_0^t v\cos\theta\,dt = v\cos\theta\,t$$

$$\int_0^y dy = \int_0^t v_y\,dt \Rightarrow y - 0 = \int_0^t (v\sin\theta - gt)\,dt = v\sin\theta\,t - \frac{1}{2}gt^2$$

When the projectile hit the incline, $\tan\phi = y/x$

$$\tan\phi = \frac{v\sin\theta t - \frac{1}{2}gt^2}{v\cos\theta t} = \tan\theta - \frac{gt}{2v\cos\theta} \Rightarrow \mathrm{t} = \frac{2v\cos\theta(\tan\theta - \tan\phi)}{g}$$

(b) Find the distance d up the incline that the projectile travels

$$x = v\cos\theta\,t = \frac{2v^2\cos^2\theta(\tan\theta - \tan\phi)}{g}$$

$$\cos\phi = \frac{x}{d} \Rightarrow \mathrm{d} = \frac{x}{\cos\phi} = \frac{2v^2\cos^2\theta(\tan\theta - \tan\phi)}{g\cos\phi}$$

(c) For what value of θ is d a maximum, and what is that maximum value?

高中方法$d = \frac{2v^2\cos^2\theta(\tan\theta - \tan\phi)}{g\cos\phi} = \frac{2v^2}{g\cos\phi}\cos^2\theta\left(\frac{\sin\theta}{\cos\theta} - \tan\phi\right) =$

$$\frac{2v^2}{g\cos\phi}(\cos\theta\sin\theta - \cos^2\theta\tan\phi) = \frac{2v^2}{g\cos\phi}\left[\frac{1}{2}\sin 2\theta - \frac{1}{2}(\cos 2\theta + 1)\tan\phi\right]$$

$$= \frac{v^2}{g\cos\phi}(\sin 2\theta - \cos 2\theta\tan\phi - \tan\phi) = \frac{v^2}{g\cos\phi}\left(\sin 2\theta - \cos 2\theta\frac{\sin\phi}{\cos\phi} - \tan\phi\right)$$

$$= \frac{v^2}{g\cos\phi}\left(\frac{\sin 2\theta\cos\phi - \cos 2\theta\sin\phi}{\cos\phi} - \tan\phi\right) = \frac{v^2}{g\cos\phi}\left(\frac{\sin(2\theta - \phi)}{\cos\phi} - \tan\phi\right) \quad \left(\theta < \frac{\pi}{2}\right)$$

当$2\theta - \phi = \frac{\pi}{2}$时，d 取最大值：$\mathrm{d}_{max} = \frac{v^2}{g\cos\phi}\left(\frac{1}{\cos\phi} - \tan\phi\right) = \frac{v^2(1 - \sin\phi)}{g\cos^2\phi}$

Exercise 1: 某自由落体$x_0 = 0$其速度-时间关系为,$v(t) = gt$，求第 1 秒时位移.

Exercise 2: Known the velocity function of a particle moving in a plan is $\vec{v}(t) = 2\vec{\imath} - gt\vec{\jmath}$, v is in m/s and $g = 9.8\,m/s^2$. At time $t = 0s$, the displacement $\vec{r_0} = 0m$. Find the displacement at *t=2s*.

Acceleration 加速度：

初等数学	高等数学
Average acceleration 平均加速度: $a_{ave} = \bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$ **Instantaneous acceleration 瞬时加速度：** $a = \lim\limits_{\Delta t \to 0} \bar{a} = \lim\limits_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$	**Average acceleration 平均加速度:** 同初等数学 **Instantaneous acceleration 瞬时加速度：** $a = \lim\limits_{\Delta t \to 0} \bar{a} = \lim\limits_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$ $dv = adt \quad or \quad \int_{v0}^{v} dv = \int_0^t adt$
Motion in 2 dimensions 二维运动: $\vec{a} = a_x \vec{i} + a_y \vec{j} = \lim\limits_{\Delta t \to 0} \vec{a}_{ave} = \lim\limits_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$	**Motion in 2 dimensions 二维运动:** $\vec{a} = a_x \vec{i} + a_y \vec{j} = \lim\limits_{\Delta t \to 0} \vec{a}_{ave}$ $= \lim\limits_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$ $\begin{cases} a_x = dv_x/dt \\ a_y = dv_y/dt \end{cases} \qquad \begin{cases} dv_x = a_x dt \\ dv_y = a_y dt \end{cases}$ $\begin{cases} \int_{vx0}^{vx} dv_x = \int_0^t a_x dt \\ \int_{vy0}^{vy} dv_y = \int_0^t a_y dt \end{cases}$

初等数学和微积分公式不要用混

初等数学	高等数学
Velocity 速度 $v(t) = v(0) + at$ 条件: a为常量	**Velocity 速度** $a = \frac{dv}{dt} \Rightarrow dv = adt$ $\int_{v0}^{v} dv = v - v_0 = \int_0^t adt$ 当 a 为常量: $v = v_0 + at$
Displacement 位移 $x(t) = x(0) + v(0)t + \frac{1}{2}at^2$ 条件: a为常量 不要用混	**Displacement 位移** $v = \frac{dx}{dt} \quad or \quad dx = vdt$ $\Rightarrow \int_{x0}^{x} dx = x - x_0 = \int_0^t v\, dt$ 当 a 为常量: $x = x_0 + \int_0^t (v(0) + at)\, dt$ $x(t) = x(0) + v(0)t + \frac{1}{2}at^2$

Example: The acceleration of a moving object is given by $a(t) = -\sin t$. At time $t = 0, x(0) = 0, v(0) = V$. Find the velocity function $v(t)$.

$$\int_V^v dv = \int_0^t a\,dt = \int_0^t -\sin t\,dt \Rightarrow v|_V^v = (\cos t)_V^v \Rightarrow v = (\cos t - 1) + V$$

Exercise 3: Find the equation for the position x of a particle whose acceleration is given by the equation $a(t) = 6t - 3$ and starts at rest from the origin.

Exercise 4: A particle moving in one dimension has a position function defined as: $x(t) = t^4 - 4t$.

(a) At what point in time does the particle change its direction along x axis?
The time the particle changing direction happens when velocity equals zero.
What is its position when it changes its direction.

(b) In what direction is the body traveling when its acceleration is 3m/s^2?

Exercise 5: A object is moving along x-axis. Its acceleration function with time t (unit in second) is $a = 4t\ m/s^2$. When $t = 0$, the object rests at 10m away from origin in positive direction of x-axis. Try to find out velocity and position function respect to time.

Exercise 6: A object is moving in a straight line at $a = dv/dt = -kv^2 t$, k is a constant number and v is the function of t. When $t = 0, v = v_0$. Find the velocity function $v(t)$.

Exercise 7: The acceleration of a particle is given by $\vec{a} = 3.0t^2\vec{i} - 2.0\vec{j}$, the initial conditions are when $t = 0, \vec{r_0} = 0, \vec{v_0} = 1.0\vec{i}$. Find its velocity and position function $\vec{v}(t)$ and $\vec{r}(t)$.

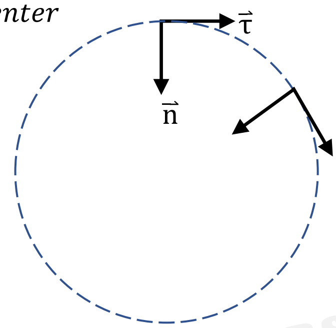
Circular Motion 圆周运动：

Unit Vector 单位向量

Tangent unit vector 切向单位向量 $\vec{\tau}\begin{cases} magnitute = 1 \\ direction\ is\ always\ at\ tangent \end{cases}$

Normal unit vector 法向单位向量 $\vec{n}\begin{cases} magnitute = 1 \\ direction\ \ point\ \ to\ \ the\ center \end{cases}$

Propriety: $\vec{\tau} \cdot \vec{\tau} = 1 \quad \vec{n} \cdot \vec{n} = 1$



线速度：永远是切线方向

$\vec{v} = v(t)\vec{\tau}$

加速度

$$\vec{a} = \frac{d\vec{v}}{dt}\begin{cases} 向心加速度\ \vec{a}_n = \frac{v^2}{r}\vec{n} \\ 切向加速度\vec{a}_t = \frac{dv}{dt}\vec{\tau} \end{cases}$$

Average angular velocity 平均角速度：

$$\bar{\omega} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t} \quad (\theta\ in\ rad)$$

Instantaneous angular velocity 瞬时角速度：

$$\omega = \lim_{\Delta t \to 0} \bar{\omega} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

$$d\theta = \omega dt$$

If the radius is r, x is the length of arc

$$x = r\theta$$

$$v = r\omega$$

$$a_n = \frac{v^2}{r} = \omega^2 r$$

Angular acceleration 角加速度：

$$\alpha = \lim_{\Delta t \to 0} \bar{\alpha} = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

Example: 对于切向单位向量$\vec{\tau}$，证明$d\vec{\tau}/dt \perp \vec{\tau}$

$$\vec{\tau}\cdot\vec{\tau} = 1 \Rightarrow \frac{d}{dt}(\vec{\tau}\cdot\vec{\tau}) = \frac{d}{dt}1 \Rightarrow \frac{d\vec{\tau}}{dt}\cdot\vec{\tau} + \vec{\tau}\cdot\frac{d\vec{\tau}}{dt} = 0$$

$$\frac{d\vec{\tau}}{dt}\cdot\vec{\tau} \neq 0 \Rightarrow \frac{d\vec{\tau}}{dt} \perp \vec{\tau}$$

Exercise 8: 已知某质点运动轨迹为半径 r 的圆，其角速度为ω，利用线速度的定义 $v = \lim_{\Delta t\to 0}\frac{\Delta x}{\Delta t} = \frac{dx}{dt}$，$x$为圆周上的位移，证明$v = r\omega$

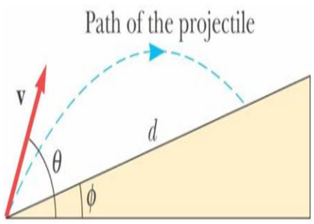
运动问题解题思路

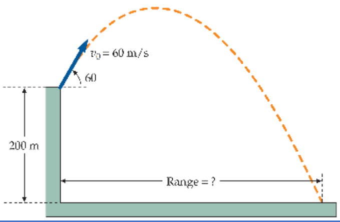
两个关键步骤：1. 判断运动类型 2.明确运动截止条件

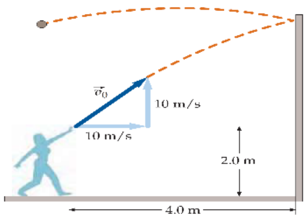
判断运动类型

$$\begin{cases} a = 0 \Rightarrow \begin{matrix}\text{constant}\\ \text{velocity}\\ \text{匀速运动}\end{matrix} \Rightarrow \begin{cases} a = 0 \\ v = c \\ x = x(0) + vt \end{cases} \\ a = c \Rightarrow \begin{matrix}\text{constant}\\ \text{acceleration}\\ \text{匀加速度}\end{matrix} \begin{cases} \xRightarrow{\text{时间已知}} \begin{cases} a = c \\ v = v(0) + at \\ x = x(0) + vt + \frac{1}{2}at^2 \end{cases} \\ \xRightarrow{\text{位移已知}} [v(t)]^2 = [v(0)]^2 + 2a\Delta x \end{cases} \\ a = ? \Rightarrow \begin{matrix}\text{Other types}\\ \text{of motion}\\ \text{其他运动}\end{matrix} \Rightarrow \begin{cases} a_{avg} = \bar{a} = \frac{\Delta v}{\Delta t} \\ a = \lim_{\Delta t\to 0}\frac{\Delta v}{\Delta t} = \frac{dv}{dt} \\ \Delta v = v(t) - v(0) \\ \Delta x = x(t) - x(0) \end{cases} \end{cases}$$

运动截止条件:







Exercise 1: 某自由落体的速度-时间关系为$v(t)=gt$，求第 1 秒时的位移.

$$\left.\begin{matrix} v = gt \\ dx = vdt = gtdt \end{matrix}\right\} \Rightarrow \int_0^x dx = \int_0^1 gt\,dt \;\Rightarrow\; x\Big|_0^x = \frac{1}{2}gt^2\Big|_0^1 = \frac{1}{2}g$$

Exercise 2: Known the velocity function of a particle moving in a plan is $\vec{v}(t)=2\vec{\imath}-gt\vec{\jmath}$, v is in m/s and $g=9.8\,m/s^2$. At time $t=0s$, the displacement $\vec{r_0}=0m$. Find the displacement at *t=2s*.

$$\int_0^x dx = \int_0^2 v_x\,dt = \int_0^2 2\,dt \Rightarrow x|_0^x = 2t|_0^2 \Rightarrow x = 4\ \ (m)$$

$$\int_0^y dy = \int_0^2 v_y\,dt = \int_0^2 -gt\,dt \Rightarrow \quad y\Big|_0^y = -\tfrac{1}{2}gt^2\Big|_0^2$$

$$y = -\tfrac{1}{2}g(4-0) = -2g = -19.6\ \ (m)$$

$$\vec{r} = x\vec{\imath} + y\vec{\jmath} = 4\vec{\imath} - 19.6\vec{\jmath}\ \ \ (m)$$

Exercise 3: Find the equation for the position x of a particle whose acceleration is given by the equation $a(t)=6t-3$ and starts at rest from the origin.

$$a(t) = 6t-3,\ v_0 = 0, x_0 = 0$$

$$\int_0^v dv = \int_0^t a\,dt = \int_0^t (6t-3)\,dt \Rightarrow v|_0^v = (6\tfrac{t^2}{2} - 3t)_0^t \Rightarrow v = 3t^2 - 3t$$

$$\int_0^x dx = \int_0^t v\,dt = \int_0^t (3t^2-3t)\,dt \Rightarrow x|_0^x = (3\tfrac{t^3}{3} - 3\tfrac{t^2}{2})_0^t \Rightarrow x = t^3 - \tfrac{3}{2}t^2$$

Exercise 4: A particle moving in one dimension has a position function defined as: $x(t)=t^4-4t$.

(a) At what point in time does the particle change its direction along x axis?
The time the particle changing direction happens when velocity equals zero.
What is its position when it changes its direction.

$$v(t) = \tfrac{dx}{dt} = \tfrac{d}{dt}(t^4-4t) = 4t^3 - 4 = 0\ \Rightarrow t = 1\ (s)$$

$$x(1) = 1 - 4 = -3\ (m)$$

(b) In what direction is the body traveling when its acceleration is $3\mathrm{m/s^2}$?

$$a(t) = \tfrac{dv}{dt} = \tfrac{d}{dt}(4t^3-4) = 12t^2 = 3\ \Rightarrow t = 0.5\ (s)$$

$$v(2) = 4\cdot(0.5)^3 - 4 = -3.5\ (m/s)$$

Exercise 5: A object is moving along x-axis. Its acceleration function with time t (unit in second) is $a=4t\ m/s^2$. When $t=0$, the object rests at 10m away from origin in positive direction of x-axis. Try to find out velocity and position function respect to time.

Known: $t=0, v(0)=0, x(0)=10, a=dv/dt=4t$.

$$\int_0^v dv = \int_0^t 4t\,dt \Rightarrow v|_0^v = \left(4\tfrac{t^2}{2}\right)_0^t \Rightarrow v = 2t^2$$

$$\int_{x0}^x dx = \int_0^t v\,dt = \int_0^t 2t^2\,dt \Rightarrow \quad x\Big|_{x0}^x = \left(\frac{2}{3}t^3\right)_0^t$$

$$\Rightarrow x - x_0 = \tfrac{2}{3}t^3 \Rightarrow x = \tfrac{2}{3}t^3 + 10$$

Exercise 6: A object is moving in a straight line at $a=dv/dt=-kv^2t$, k is a constant number and v is the function of *t*. When $t=0, v=v_0$. Find the velocity function $\mathrm{v}(t)$.

$$\frac{dv}{dt} = -kv^2t\ \Rightarrow \int_{v0}^v v^{-2}dv = \int_0^t -ktdt \Rightarrow \frac{v^{-1}}{-1}\Big|_{v0}^v = -k\left(\frac{t^2}{2}\right)_0^t$$

$\frac{1}{v} = \frac{1}{2}kt^2 + \frac{1}{v_0}$

特别注意$v_0 \neq 0$时这种初始条件

Exercise 7: The acceleration of a particle is given by $\vec{a} = 3.0t^2\vec{i} - 2.0\vec{j}$, the initial conditions are when $t = 0, \vec{r_0} = 0, \vec{v_0} = 1.0\vec{i}$. Find its velocity and position function $\vec{v}(t)$ and $\vec{r}(t)$.

$$\int_{v0}^{v} d\vec{v} = \int_0^t \vec{a}dt = \int_0^t \left(3.0t^2\vec{i} - 2.0\vec{j}\right)dt \Rightarrow \vec{v} - \vec{v}_0 = 3\frac{t^3}{3}\vec{i} - 2.0t\vec{j}$$

$$\Rightarrow \vec{v} = \vec{v}(t) = (t^3 + 1.0)\vec{i} - 2.0t\vec{j}$$

$$\int_0^r d\vec{r} = \int_0^t \vec{v}dt = \int_0^t [(t^3 + 1.0)\vec{i} - 2.0t\vec{j}]dt$$

$$\Rightarrow \vec{r} = \vec{r}(t) = \left(\frac{t^4}{4} + t\right)\vec{i} - t^2\vec{j}$$

Exercise 8: Known that the trajectory of a particle is a circle with radius r and its angular velocity is ω, using the definition of linear velocity, prove that $v = r\omega$.

$$\left.\begin{matrix} v=\frac{dx}{dt} \\ x=r\theta \end{matrix}\right\} \Rightarrow v = \frac{dr\theta}{dt} = r\frac{d\theta}{dt} = r\omega$$